where μ = gas viscosity, lb/ft-s
B_c = cyclone inlet width, ft
n_t = number of turns
v_i = inlet gas velocity, ft/s
ρ_p = particle density, lb/ft^3
ρ = gas density, lb/ft^3

1.a.1. Determine the inlet width of the cyclone, B_c.

The permit application has established this cyclone as conventional. The inlet width of a conventional cyclone is 1/4 the cyclone diameter.

$$B_c = \text{cyclone diameter}/4 = 2.0/4 = 0.5\,\text{ft}$$

1.a.2. Determine the value of $\rho_p - \rho$.

Since the particle density is much greater than the gas density, $\rho_p - \rho$ can be assumed to be ρ_p.

$$\rho_p - \rho = \rho_p = 2.75(62.4) = 171.6\,\text{lb/ft}^3$$

1.a.3. Calculate the cut diameter using the equation given.

$$\begin{aligned}[d_p]_{\text{cut}} &= [9\mu B_c/2\pi n_t v_i(\rho_p - \rho)]^{0.5} \\ &= [(9)(1.21 \times 10^{-5})(0.5)/(2\pi)4.5(50)(171.6)]^{0.5} \\ &= 1.5 \times 10^{-5}\,\text{ft} \\ &= 4.57\,\text{microns}\end{aligned}$$

1.b. Calculate the ratio of average particle diameter to the cut diameter.

$$d_p/[d_p]_{\text{cut}} = 7.5/4.57 = 1.64$$

1.c. Determine the collection efficiency utilizing Lapple's curve (see Fig. 2.26).

$$\eta = 72\%$$

2. Calculate the required collection efficiency for the approval of the permit.

$$\begin{aligned}\eta &= [(\text{inlet loading} - \text{outlet loading})/(\text{inlet loading})](100) \\ &= [(0.5 - 0.1)/(0.5)](100) \\ &= 80\%\end{aligned}$$

3. Would you approve the permit?

Since the collection efficiency of the cyclone is lower than the collection efficiency required by the agency, the permit should not be approved.

4.3 Electrostatic Precipitator (ESP)

An electrostatic precipitator (ESP) is an effective device for controlling particle emissions from cement kiln, pulp and paper plants, acid plants, sintering operations, and other industrial sources. The method is extensively used where dust emissions are less than 10–20 μm in size with a predominant portion in the submicron range (EPA-81/10, p. 7-1).

In general, an electrostatic precipitator is comprised of four essential components, as shown in Fig. 2.27. The essential components are

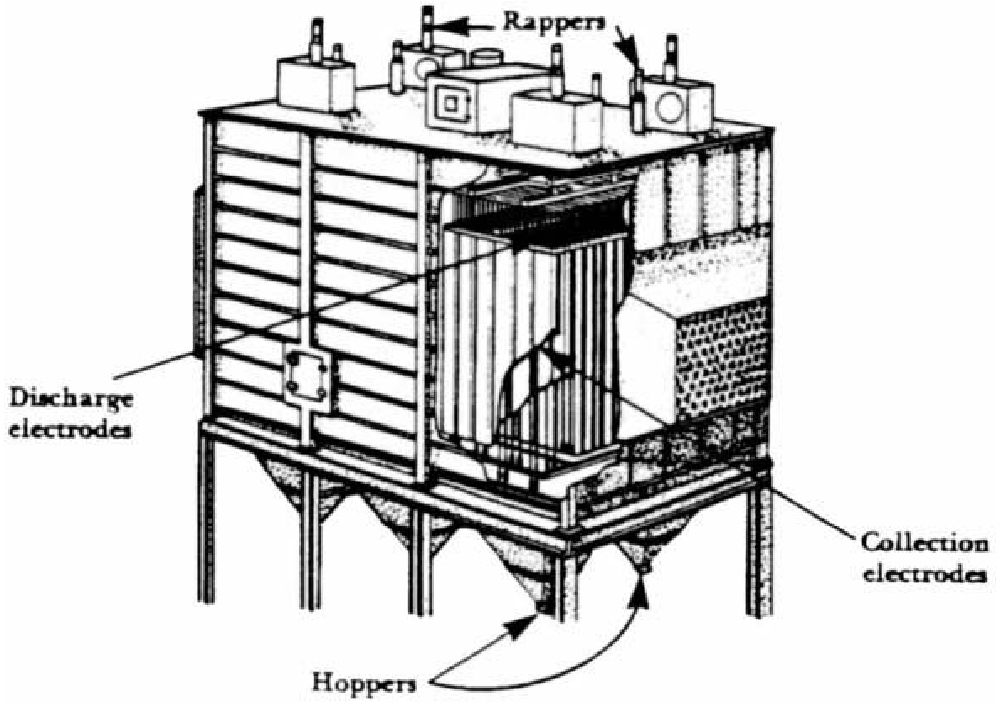

FIGURE 2.27 Typical plate and wire single-stage ESP (*EPA-81/10, p. 7-1*).

- discharge electrodes
- collection electrodes
- rappers
- hoppers

The discharge electrode is normally a wire where a corona discharge occurs. This electrode is used to ionize the gas (which charges the particles) and create an electric field. The collection electrode consists of either a tube or flat plate which is oppositely charged (relative to the discharge electrode) and is the surface where the charged particles are collected. The rapper is a device used to impart a vibration or shock to dislodge the deposited dust on the electrodes. Rappers are used to remove dust accumulated on both the collection electrodes and discharge electrodes. Hoppers are located at the bottom of the precipitator and are used to collect and store the dust removed by the rapping process.

ESP type. The types of electrostatic precipitators include (EPA-81/10, p. 7-2):

- Low voltage two-stage precipitators
- High voltage single-stage precipitators
    - Tubular high voltage single-stage precipitators;
    - Plate high voltage single-stage precipitators

Low voltage two-stage ESPs: Low voltage two-stage precipitators are limited almost exclusively to the collection of liquid aerosols discharged from sources such as meat smokehouses, pipe-coating machines, asphalt paper saturators, and high-speed grinding machines. The precipitators were originally designed for air purification in conjunction with air conditioning systems (they are also referred to as electronic air filters). Two-stage ESPs have been used primarily for the control of finely divided liquid particles. Controlling solid or sticky materials is usually difficult, and the collector becomes ineffective for dust loadings greater than 0.4

grains per standard cubic foot (0.916 g/m^3). Therefore, two-stage precipitators have limited use for particulate emission control.

High voltage single-stage precipitators: The high voltage single-stage precipitator is the more popular type and has been used successfully to collect both solid and liquid particulate matter in industrial facilities such as smelters, steel furnaces, cement kilns, municipal incinerators, and utility boilers. There are two major types of high voltage single-stage ESP configuration. Particles are both charged and collected in a single stage.

- *Tubular precipitators*: Tubular precipitators consist of cylindrical collection electrodes with discharge electrodes located in the center of the cylinders. Dirty gas flows into the cylinder, where precipitation occurs. The negatively charged particles migrate to and are collected on grounded collecting tubes. The collected dust or liquid is removed by washing the tubes with water sprays located directly above the tubes. These precipitators are generally referred to as water-washed ESPs. Tubular precipitators are generally used for collecting mists or fogs. Tube diameters typically vary from 0.5 to 1 ft (0.15 to 0.31 m), with length usually ranging from 6 to 15 ft (1.85 to 4.6 m).
- *Plate precipitators*: Plate electrostatic precipitators are used more often than tubular ESPs in industrial applications. High voltage is used to subject the particles in the gas stream to an intense electric field. Dirty gas flows into a chamber consisting of a series of discharge electrodes (wires) spaced along the center line of adjacent plates, as shown in Fig. 2.28. Charged particles migrate to and are collected at oppositely charged collection plates. Collected particles are usually removed by rapping (dry precipitator) or by a liquid film (wet precipitator). Particles fall by force of gravity into hoppers, where they are stored prior to removal and final disposal.

Collection efficiency. ESP collection efficiency can be expressed by the following two equations (EPA-81/10, p. 7-9):

- Migration velocity equation
- Deutsch–Anderson equation

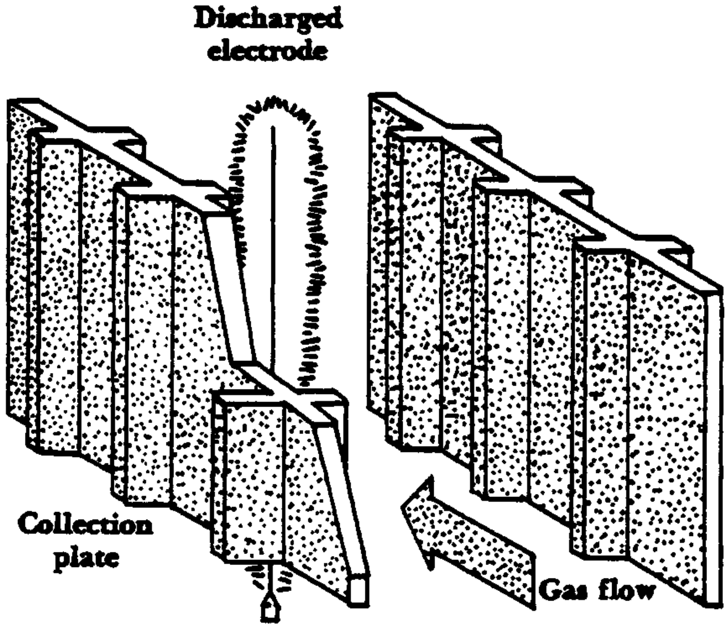

FIGURE 2.28 Gas flow through a wire and plate precipitator (*EPA-81/10, p. 7-4*).

Migration velocity: Once the particle is charged, it migrates toward the grounded collection electrode. An indicator of particle movement toward the collection electrode is denoted by the symbol w and is called the *particle migration velocity* or *drift velocity*. The migration velocity parameter represents the collectability of the particle within the confines of a specific collector. The migration velocity can be expressed in terms of

$$w = d_p E_o E_p / (4\pi\mu) \tag{2.11}$$

where w = migration velocity
d_p = diameter of the particle, μm
E_o = strength of field in which particles are charged, volts per meter (represented by peak voltage)
E_p = strength of field in which particles are collected, volts per meter (normally the field close to the collecting plates)
μ = viscosity of gas, Pa-s

Migration velocity is quite sensitive to the voltage, since the electric field appears twice in Eq. (2.11). Therefore, the precipitator must be designed using the maximum electric field for maximum collection efficiency. The migration velocity is also dependent on particle size; larger particles are collected more easily than smaller ones.

Particle migration velocity can also be determined by the following equation:

$$w = qE_p / (4\pi\mu r) \tag{2.12}$$

where w = migration velocity
q = particle charge (charges)
E_p = strength of field in which particles are collected, volts per meter (normally the field close to the collecting plates)
μ = viscosity of gas, Pa-s
r = radius of the particle, μm

Deutsch–Anderson equation: This equation has been used to determine the collection efficiency of the precipitator under ideal conditions. The simplest form of the equation is

$$\eta = 1 - \exp(-wA/Q) \tag{2.13}$$

where η = fractional collection efficiency
A = collection surface area of the plates
Q = gas volumetric flow rate
w = drift velocity

This equation has been used extensively for many years for theoretical collection efficiency calculations. Unfortunately, while the equation is scientifically valid, there are a number of operating parameters that can cause the results to be in error by a factor of two or more. The Deutsch–Anderson equation neglects three significant process variables.

1. It completely ignores the fact that dust re-entrainment may occur during the rapping process.
2. It assumes that the particle size and, consequently, the migration velocity is uniform for all particles in the gas stream.
3. It assumes that the gas flow rate is uniform everywhere across the precipitator and that particle sneakage through the hopper section does not occur.

Therefore, this equation should be used only for making preliminary estimates of precipitation collection efficiency.

Design parameters. Many parameters must be taken into consideration in the design and specification of electrostatic precipitators. The typical design parameters include (EPA-81/10, p. 7-11):

- Resistivity
- Specific collection area
- Aspect ratio
- Gas flow distribution
- Electrical sectionalization

Resistivity: Particle resistivity is a condition of the particle in the gas stream that can alter the actual collection efficiency of an ESP design. Resistivity is a term that describes the resistance of the collected dust layer to the flow of electrical current. By definition, *resistivity* is the electrical resistance of a dust sample $1.0\,\mathrm{cm}^2$ in cross-sectional area, 1.0 cm thick; it is recorded in units of ohm-cm. It can also be described as the resistance to charge transfer by the dust. Dust resistivity values can be classified roughly into three groups:

- between 10^4 and 10^7 ohm-cm (low resistivity)
- between 10^7 and 10^{10} ohm-cm (normal resistivity)
- above 10^{10} ohm-cm (high resistivity)

Specific collection area (SCA): The specific collection area is defined as the ratio of collection surface area to the gas flow rate into the collector. The importance of this term is that it represents the A/Q relationship in the Deutsch–Anderson equation.

$$\begin{aligned}\text{SCA} &= (\text{Total collection surface, ft}^2)/[\text{flow rate (1000 acfm)}]\\ &= \text{m}^2/(1000\,\text{m}^3/\text{h}) \text{ in metric units}\end{aligned}$$

Increases in the SCA of a precipitator design will in most cases increase the collection efficiency of the precipitator. Most conservative designs call for an SCA of 350 to 400 ft^2 per 1000 acfm (19 and 22 m^2 per 1000 m^3/h) to achieve 99.5% particle removal. The general range of SCA is between 200 and 800 ft^2 per 1000 acfm (11 and 45 m^2 per 1000 m^3/h), depending on precipitator design conditions and desired collection efficiency.

Aspect ratio: The aspect ratio is the ratio of the total length to height of collector surface. The aspect ratio can be calculated by

$$\text{AR} = (\text{effective length})/(\text{effective height})$$

Having a precipitator chamber many times larger in length than in height would be ideal. However, space limitations and cost could be prohibitive. The aspect ratio for ESPs can range from 0.5 to 2.0. For 99.5% collection efficiency, the precipitator design should have an aspect ratio of greater than 1.0.

Gas flow distribution: Gas flow through the ESP chamber should be slow and evenly distributed throughout the unit. The gas velocities in the duct ahead of the ESP are generally between 20 and 80 ft/s (6 and 24 m/s). The gas velocity into the ESP must be reduced for adequate particle collection. This is achieved by using an expansion inlet plenum.

The inlet plenum contains diffuser-perforated plate openings to evenly distribute the gas flow throughout the precipitator. Typical gas velocities in the ESP chamber range from 2 to 8 ft/s (0.6 to 2.4 m/s). With aspect ratios of 1.5, the optimum gas velocity is generally between 5 and 6 ft/s (1.5 and 1.8 m/s).

Electrical sectionalization: Precipitator performance is dependent on the number of individual sections or fields installed. The maximum voltage at which a given field can be maintained depends on the properties of the gas and dust being collected. These parameters may vary from one point to another in the unit. To keep each section of the precipitator working at high efficiency, a high degree of sectionalization is recommended. Multiple fields or stages are used to provide electrical sectionalization. Each field has separate power supplies and controls to adjust for varying gas conditions in the unit.

In general, precipitators have voltage control devices that automatically limit precipitator power input. A well-designed automatic control system keeps the voltage level at approximately the value needed for optimum particle charging by the discharge electrodes.

The voltage control devices operate in the following manner: increases in voltage cause a greater spark rate between the discharge and collection electrodes. Occurrence of a spark counteracts high ESP performance, since it causes an immediate, short-term collapse of the precipitator field. Consequently, less useful power is applied to capture particles. There is, however, an optimal sparking rate where the gains in particle charging are just offset by corona current losses from sparkover.

Measurements on commercial precipitators have determined that the optimal sparking rate is between 50 and 150 sparks per minute per electrical section. The objective in power control is to maintain corona power input at this optimal sparking rate. This can be accomplished by momentarily reducing precipitator power whenever excessive sparking occurs.

The need for separate fields arises mainly because power input requirements differ at various locations in a precipitator. The particulate matter concentration is generally high at the inlet sections of the precipitator. High dust concentrations tend to suppress corona current. Therefore, a great deal of power is needed to generate corona discharge for optimal particle charging at the inlet.

In the downstream fields of a precipitator, the dust loading is usually lighter. Consequently, corona current flows freer in downstream fields. Particle charging will more likely be limited by excessive sparking in downstream fields than in the inlet fields. The power to the outlet sections must still be high in order to collect small particles, particularly if they exhibit high resistivity.

If the precipitator had only one power set, the excessive sparking would limit the power input to the entire precipitator. This would result in a reduction of overall collection efficiency.

Review of ESP design plan. The first step in reviewing design plans for air pollution permits is to read the vendor literature and specifications of the precipitator design. The design specifications should include at least (EPA-81/10, p. 7-26):

- Exhaust gas flow rate and temperature
- Inlet dust concentration
- Specific collection area (SCA)
- Gas velocity in the precipitator
- Distance between the plates
- Aspect ratio
- Number and size of transformer-rectifier (T-R) sets
- Number of fields
- Design migration velocity
- Corona power/1000 m^3/min
- Corona current/ft^2 plate area
- Design collection efficiency
- Outlet dust concentration

The next step is to review the outlet concentration from the ESP. The concentration must meet the grain-loading requirements of air pollution regulations. The design reviewer can determine if the calculated outlet values, using the Deutsch–Anderson equation, are within the regulation limits. In addition, requiring the source to perform a source test to verify the designed collection efficiency of the ESP would be extremely useful.

EXAMPLE 1: electrostatic precipitator—process change. A horizontal parallel-plate electrostatic precipitator consists of a single duct 24 ft high and 20 ft deep with an 11 inch plate-to-plate spacing. Given a collection efficiency at a gas flow rate of 4200 acfm, you are required to determine the bulk velocity of the gas, outlet loading, and drift velocity of this electrostatic precipitator. You are also requested to calculate a revised collection efficiency if the flow rate and the plate spacing are changed (EPA-84/09, p. 71).

Given conditions

inlet loading $= 2.82\,\text{grains/ft}^3$
collection efficiency at 4200 acfm = 88.2%
increased (new) flow rate = 5400 acfm
new plate spacing = 9 in.

Solution:

1. Calculate the bulk flow (throughput) velocity v.
The equation for calculating throughput velocity is

$$V = Q/S$$

where Q = gas volumetric flow rate
S = cross-sectional area through which the gas passes

$$\begin{aligned} V &= Q/S \\ &= (4200)/[(11/12)(24)] \\ &= 191\ \text{ft/min} \\ &= 3.2\ \text{ft/s} \end{aligned}$$

2. Calculate outlet loading
Remember that

$$\eta\ (\text{fractional}) = (\text{inlet loading} - \text{outlet loading})/(\text{inlet loading})$$

Therefore

$$\begin{aligned} \text{Outlet loading} &= (\text{inlet loading})(1 - \eta) \\ &= (2.82)(1 - 0.882) \\ &= 0.333\ \text{grains/ft}^3 \end{aligned}$$

3. Calculate the drift velocity.
The drift velocity is the velocity at which the particle migrates toward the collection electrode within the electrostatic precipitator.
The Deutsch–Anderson equation describing the collection efficiency of an electrostatic precipitator is

$$\eta = 1 - \exp(-wA/Q)$$

where η = fractional collection efficiency
A = collection surface area of the plates
Q = gas volumetric flow rate
w = drift velocity

3.a. Calculate the collection surface area A.

Remember that the particles will be collected on both sides of the plate.

$$A = (2)(24)(20) = 960\,\text{ft}^2$$

3.b. Calculate the drift velocity w.

Since the collection efficiency, gas flow rate, and collection surface area are now known, the drift velocity can easily be found from the Deutsch–Anderson equation:

$$\eta = 1 - \exp(-wA/Q)$$
$$0.882 = 1 - \exp[-(960)(w)/(4200)]$$

Solving for w

$$w = 9.36\,\text{ft/min}$$

4. Calculate the revised collection efficiency when the gas volumetric flow rate is increased to 5400 acfm.

Assume the drift velocity remains the same.

$$\begin{aligned}\eta &= 1 - \exp(-wA/Q)\\ &= 1 - \exp(-(960)(9.36)/(5400)]\\ &= 0.812\\ &= 81.2\%\end{aligned}$$

5. Does the collection efficiency change with changed plate spacing?

No. Note that the Deutsch–Anderson equation does not contain a plate-spacing term.

EXAMPLE 2: electrostatic precipitator—collection efficiency. You have been requested to calculate the collection efficiency of an electrostatic precipitator containing three ducts with plates of a given size, assuming a uniform distribution of particles. Also, determine the collection efficiency if one duct is fed 50% of the gas and the other passages 25% each (EPA-84/09, p. 73).

Given conditions

volumetric flow rate of contaminated gas = 4000 acfm

operating temperature and pressure = 200°C and 1 atm

drift velocity = 0.40 ft/s

size of the plate = 12 ft long and 12 ft high

plate-to-plate spacing = 8 in.

Solution:

1. What is the collection efficiency of the electrostatic precipitator with a uniform volumetric flow rate to each duct?

The Deutsch–Anderson equation describing the collection efficiency of an electrostatic precipitator is

$$\eta = 1 - \exp(-wA/Q)$$

where η = fractional collection efficiency
A = collection surface area of the plates

Q = gas volumetric flow rate
w = drift velocity

1.a. Calculate the collection surface area per duct, A.
Considering both sides of the plate,

$$A = (2)(12)(12) = 288\,\text{ft}^2$$

1.b. Calculate the collection efficiency of the electrostatic precipitator using the Deutsch–Anderson equation.

The volumetric flow rate (Q) through a passage is one-third of the total volumetric flow rate,

$$\begin{aligned} Q &= (4000)/(3)(60) \\ &= 22.22\,\text{acfs} \\ \eta &= 1 - \exp(-wA/Q) \\ &= 1 - \exp[-(288)(0.4)/(22.22)] \\ &= 0.9944 \\ &= 99.44\% \end{aligned}$$

2. What is the collection efficiency of the electrostatic precipitator if one duct is fed 50% of gas and the others 25% each. The collection surface area per duct remains the same.

2.a. What is the collection efficiency of the duct with 50% of gas, η_1?

2.a.1. Calculate the volumetric flow rate of gas through the duct in acfts.

$$Q = (4000)/(2)(60) = 33.33\,\text{acfs}$$

2.a.2. Calculate the collection efficiency of the duct with 50% of gas.

$$\begin{aligned} \eta_1 &= 1 - \exp[-(288)(0.4)/(33.33)] \\ &= 0.9684 \\ &= 96.84\% \end{aligned}$$

2.b. What is the collection efficiency (η_2) of the ducts with 25% of gas flow in each?
2.b.1. Calculate the volumetric flow rate of gas through the duct in acfs.

$$Q = (4000)/(4)(60) = 16.67\,\text{acfs}$$

2.b.2. Calculate the collection efficiency (η_2) of the duct with 25% of gas.

$$\begin{aligned} \eta_2 &= 1 - \exp[-(288)(0.4)/(16.67)] \\ &= 0.9990 \\ &= 99.90\% \end{aligned}$$

2.c. Calculate the new overall collection efficiency.
The key equation becomes:

$$\begin{aligned} \eta_t &= (0.5)(\eta_1) + (2)(0.25)(\eta_2) \\ &= (0.5)(96.84) + (2)(0.25)(99.90) \\ &= 98.37\% \end{aligned}$$

EXAMPLE 3: electrostatic precipitator—plan review. Fractional efficiency curves describing the performance of a specific model of an electrostatic precipitator have been compiled by a vendor. Although you do not possess these curves, the cut diameter is known. The vendor claims that this particular model will perform with a given efficiency under your operating condition. You are asked to verify this claim and to make certain that the effluent loading does not exceed the standard set by EPA (EPA-84/09, p. 75).

Given conditions

plate-to-plate spacing = 10 in.
cut diameter = 0.9 μm (microns)
collection efficiency claimed by the vendor = 98%
inlet loading = 14 grains/ft^3
EPA standard for the outlet loading = 0.2 grains/ft^3 (maximum)

The particle size distribution is given in Table 2.15.
A Deutsch–Anderson type of equation describing the collection efficiency of an electrostatic precipitator is

$$\eta = 1 - \exp(-Kd_p)$$

where
η = fractional collection efficiency
K = empirical constant
d_p = particle diameter

TABLE 2.15 Particle Size Distribution

Weight range	Average particle size d_p, μm
0–20%	3.5
20–40%	8
40–60%	13
60–80%	19
80–100%	45

Solution:

1. Is the overall efficiency of the electrostatic precipitator equal to or greater than 98%? Since the weight fractions are given, collection efficiencies of each particle size are needed to calculate the overall collection efficiency.

1.a. Determine the value of K by using the given cut diameter.

Remember that the cut diameter is the particle diameter collected at 50% efficiency. Since the cut diameter is known, you can solve the Deutsch–Anderson type equation directly for K.

$$\eta = 1 - \exp(-Kd_p)$$
$$0.5 = 1 - \exp[-K(0.9)]$$

Solving for K,

$$K = 0.77$$

1.b. Calculate the collection efficiency using the Deutsch–Anderson equation.
Use the Deutsch–Anderson equation to calculate the collection efficiency. For $d_p = 3.5$

$$\begin{aligned}\eta &= 1 - \exp[(-0.77)(3.5)]\\ &= 0.9325\end{aligned}$$

Table 2.16 shows the collection efficiency for each particle size.

TABLE 2.16 Collection Efficiency for Each Particle Size

Weight fraction w_i	Average particle size d_p, μm	η_i
0.2	3.5	0.9325
0.2	8	0.9979
0.2	13	0.9999
0.2	19	0.9999
0.2	45	0.9999

1.c. Calculate the overall collection efficiency.

$$\begin{aligned}\eta &= \Sigma w_i \eta_i\\ &= (0.2)(0.9325) + (0.2)(0.9979) + (0.2)(0.9999) + (0.2)(0.9999) + (0.2)(0.9999)\\ &= 0.9861\\ &= 98.61\%\end{aligned}$$

where η = overall collection efficiency
w_i = weight fraction of ith particle size
η_i = collection efficiency of ith particle size

1.d. Is the overall collection efficiency greater than 98%?
Yes

2. Does the outlet loading meet EPA's standard?

2.a. Calculate the outlet loading in grains/ft^3.

$$\text{Outlet loading} = (1.0 - \eta)(\text{inlet loading})$$

where η is the fractional efficiency for the above equation.

$$\begin{aligned}\text{Outlet loading} &= (1.0 - 0.9861)(14)\\ &= 0.195\,\text{grains/ft}^3\end{aligned}$$

2.b. Is the outlet loading less than 0.2 grains/ft^3.
Yes

3. Is the vendor's claim verified?
Yes.

4.4 Fabric Filtration

Fabric filtration is one of the most common techniques used to collect particulate matter (EPA-81/10, p. 8-1).